

Designing a PI controller for Cuk converter using converter dynamics toolbox for MATLAB



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ABSTRACT

This paper dedicated to study indirect control of Cuk converter. Cuk converter has a 4th order non minimum phase transfer function. Extraction of converter's dynamical equation is not an easy task with pencil-and-paper analysis. Converter's dynamical equations are obtained using "Kocaeli university's converter dynamics toolbox for MATLAB®" developed in Kocaeli university's power electronics research group. Presence of Right Half Plane (RHP) zeros force us to use two feedback loops. Controllers are designed for this two loops using MATLAB's control system toolbox. Close loop system is tested in Simulink® environment. Simulation results showed the performance of designed controller.

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1. Introduction

Power electronics converters require negative feedback to provide a suitable output voltage or current for the load. Obtaining a stable output voltage or current in presence of disturbances like: input voltage's changes and /or output load's changes seems impossible without some form of control. Although control engineering has considerable progress over recent decades, most applications use PID controllers, because of their low price and simplicity. Generally speaking, using derivative term is not so common in power electronics converters control. Usually a P or PI controller is all that is required. Designing a classical P or PI controller for a power electronics converter is started by obtaining the model of converter. Modeling is the process of formulating a mathematical description of the system. Obtaining the mathematical model of system is the first step toward designing a controller in model base controller design techniques. Switching power converters are nonlinear variable structure systems. Various techniques can be found in literature to obtain a linear continuous Time Invariant (LTI) model of a DC-DC converter. The most well-known methods are: Current injected approach, circuit

averaging and state space averaging (Middlebrook and Cuk, 1977; Kislovski et al., 1991; Mohan and Undeland, 2007; Voperian, 1990). Averaging and small signal linearization is key steps of these methods.

State space averaging described in Middlebrook and Cuk (1977) is appropriate to describe converters that work in CCM while is less suitable for converters work in DCM. The current injected method Kislovski et al. (1991) and Mohan and Undeland (2007) can do the job of modeling in either CCM or DCM. Circuit averaging gained a lot of attention recently due to its generality (Hren and Slibar, 2005).

At the time of this writing, there is no software to extract algebraic transfer function of power electronics converters. Using available commercial software only a frequency response plot can be obtained. No information about location of poles and zeros are given by software so a pencil-and-paper analysis is required to obtain the algebraic transfer function of converter. Obtaining the dynamical equation of converter in presence of circuit's non idealities such as Equivalent Series Resistance (ESR) of capacitors and inductors, voltage drop of diodes, on resistance of MOSFETS is a tedious, time consuming and error prone task for pencil-and-paper analysis. If a parameter is changed, i.e. load, all the calculation must be done from scratch. Developed software can be used to extract dynamical equation of buck, boost, buck-boost, Cuk, SEPIC, fly back, forward and full bridge converters in presence of mentioned non-idealities. Developed software can

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give both algebraic transfer function and frequency response plot of converter. Process of controller design can be started after converter's dynamical equation extraction; MATLAB's control system toolbox can be quite helpful for this phase. This paper shows how a control system can be designed for a Cuk converter. Cuk converter has a 4th order, i.e. has 4 poles, non-minimum phase transfer function, i.e. zeros are in RHP. It can be shown by Bode theorem that presence of RHP zeros degrades the achievable close loop performance. Presence of RHP zeros force us to use both current and voltage feedback. MATLAB's control system toolbox is used to tune these two loops. Proposed method can be used to design controller for other non-minimum phase converters like SEPIC.

2. Related works

Foundation of State Space Averaging (SSA) was laid down in Middlebrook and Cuk (1977). The first attempt to model Discontinuous Conduction Mode (DCM) is presented in Cuk and Middlebrook (1977). Accurate small signal models for DCM operation were developed by Sun et al. (2001). A unified SSA based method to develop both CCM and DCM was developed by Suntio (2006). A comprehensive survey of the modeling issues can be found in Maksimovic et al. (2001). Application of different control methods to power electronics converters has been studied in many papers. For example, feedback linearization (Sanders et al., 1986), sliding mode control (Sira-Ramirez, 1987), PID control (Venkatanarayanan and Saravanan, 2014) and H_{∞} design (Rodriguez et al., 2005) has been applied to Cuk converter, Linear Matrix Inequality(LMI) control has been applied to conventional boost by Reddy et al. (2015). Discrete time controller has been designed for a boost converter in Alkrunz and Yazici (2016). A cascade state space controller is designed for buck mode of bidirectional dc-dc converter in Ocilka and Béréš (2010). PID control of SEPIC converter is studied in Veenalakshmi et al. (2014).

3. Working principle of Cuk converter

Topology of Cuk converter is shown in Fig. 1. Output voltage can be either smaller or larger than that of input. There is a polarity reversal on the output. Energy transfer from input source to load depends on the capacitor C₁. Analysis of this circuit is based on the following assumptions:

1. Both inductors (capacitors) are large enough so current in (voltage across) them are constant.
2. Circuit operating in steady state, i.e. transients has been passed.
3. Switch and diode are ideal (i.e. no resistance and voltage drop).
4. Switch is closed for time DT and open for (1-D) T. Corresponding circuit for switch closed and opened is shown in Figs. 2 and 3, respectively.

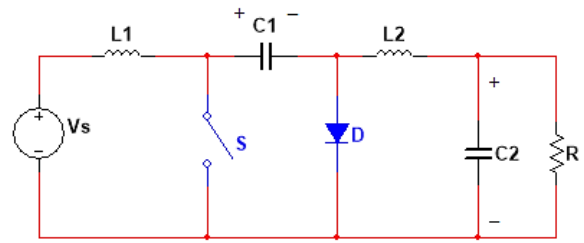


Fig. 1: Topology of Cuk converter

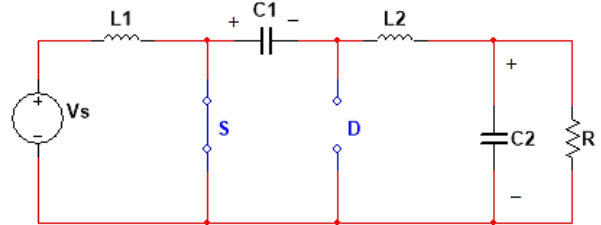


Fig. 2: Cuk converter's equivalent circuit for closed switch

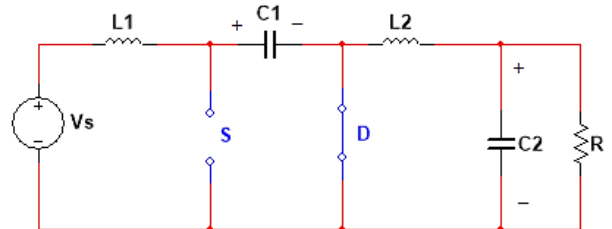


Fig. 3: Cuk converter's equivalent circuit for opened switch

Applying volt-second balance (Mohan and Undeland, 2007) to inductors lead to:

$$V_{c1} = \frac{1}{1-D} V_s \tag{1}$$

$$V_{c2} = -\frac{D}{1-D} V_s \tag{2}$$

Assuming an ideal capacitor, i.e. no Equivalent Series Resistance (ESR), output voltage ripple can be found as:

$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8L_2C_2f^2} V_s \tag{3}$$

For Continuous Current Mode (CCM) operation of Cuk converter, the inductors average current must be greater than one half the changes in current. This leads to:

$$L_{1,min} = \frac{(1-D)^2 R}{2Df} \tag{4}$$

$$L_{2,min} = \frac{(1-D)R}{2f} \tag{5}$$

converter works in CCM if $L_1 > L_{1,min}$ and $L_2 > L_{2,min}$.

4. Small signal model extraction

Assume a general state space model of following form for a CCM converter:

$$\begin{cases} \dot{x} = A_1 x + B_1 u \\ v_o = C_1^T x \end{cases} \tag{6}$$

$$\begin{cases} \dot{x} = A_2x + B_2u \\ v_o = C_2^T x \end{cases} \quad (7)$$

Eqs. 6 and 7 are written under switch close and switch open condition, respectively. x is state vector, i.e. capacitors' voltage and inductors' current, u is control input and v_o is output voltage of converter. Applying the averaging and perturbation to these equations leads to:

$$\frac{d\tilde{x}}{dt} = [A_1D + A_2(1 - D)]\tilde{x} + [(A_1 - A_2)X + (B_1 - B_2)V]\tilde{d} \quad (8)$$

$$\tilde{v}_o = [C_1^T + C_2^T(1 - D)]\tilde{x} + [(C_1^T - C_2^T)X]\tilde{d} \quad (9)$$

Tilde ($\tilde{x}, \tilde{d}, \tilde{v}_o$) shows small signal variables and D shows steady state duty ratio. Switch close's and switch open's equations for Cuk converter in presence of non-idealities such as capacitor's and inductor's ESR, MOSFET's on resistance and voltage drop of diode, can be written as:

$$\begin{aligned} \frac{d}{dt} i_{L1} &= -\frac{1}{L_1}(r_i + r_{L1} + r_{sw})i_{L1} + \frac{r_{sw}}{L_1}i_{L2} + \frac{1}{L_1}v_s \\ \frac{d}{dt} i_{L2} &= \frac{r_{sw}}{L_2}i_{L1} - \frac{r_{C1} + r_{L2} + r_{sw}}{L_2}i_{L2} - \frac{1}{L_2}v_{C1} - \frac{1}{L_2}v_{C2} \\ \frac{d}{dt} v_{C1} &= -\frac{1}{C_1}i_{L2} \\ \frac{d}{dt} v_{C2} &= \frac{1}{C_2}i_{L2} - \frac{1}{R_L \times C_2}v_{C2} \end{aligned} \quad (10)$$

for switch closed and

$$\begin{aligned} \frac{d}{dt} i_{L1} &= -\frac{1}{L_1}(r_D + r_i + r_{L1} + r_{C1})i_{L1} + \frac{r_D}{L_1}i_{L2} - \frac{1}{L_1}v_{C1} + \frac{1}{L_1}v_s - \frac{1}{L_1}v_\gamma \\ \frac{d}{dt} i_{L2} &= \frac{r_D}{L_2}i_{L1} - \frac{r_D + r_{L2} + r_{C2}}{L_2}i_{L2} - \frac{1}{L_2}v_{C2} + \frac{1}{L_2}v_\gamma \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{d}{dt} v_{C1} &= \frac{1}{C_1}i_{L1} \\ \frac{d}{dt} v_{C2} &= \frac{1}{C_2}i_{L2} - \frac{1}{R_L \times C_2}v_{C2} \end{aligned}$$

for switch opened.

Using pencil-and-paper analysis to obtain dynamical equation of this 4th order converter is a tedious, time consuming and error prone task. A toolbox is developed for MATLAB to do the SSA procedure automatically. All what must be done by user is entering converter's parameters. Assume a Cuk converter with the following values:

$V_{in}=12V$, $r_{internal}=0.6\Omega$, $L_1=500\mu H$, $L_2=700\mu H$, $r_{L1}=10m\Omega$, $r_{L2}=21m\Omega$, $C_1=20\mu F$, $C_2=4\mu F$, $r_{C1}=10m\Omega$, $r_{C2}=5m\Omega$, $V_{Diode,on}=0.7$, $r_{Diode,on}=0.05\Omega$, $r_{MOSFET}=40m\Omega$, $R_{Load}=8.1\Omega$.

After entering these values to software (Fig. 4), following results are obtained:

$$\frac{v_{C2}(s)}{d(s)} = \frac{-9.09 \times 10^9 \times s^2 + 3.618 \times 10^{13} \times s - 2.992 \times 10^{17}}{s^4 + 3.228 \times 10^4 \times s^3 + 4.428 \times 10^8 \times s^2 + 1.8 \times 10^{12} \times s + 6.903 \times 10^{15}} \quad (12)$$

$$\frac{i_{L1}(s)}{d(s)} = \frac{5.103 \times 10^4 \times s^3 + 1.754 \times 10^9 \times s^2 + 2.597 \times 10^{13} \times s + 1.317 \times 10^{17}}{s^4 + 3.228 \times 10^4 \times s^3 + 4.428 \times 10^8 \times s^2 + 1.8 \times 10^{12} \times s + 6.903 \times 10^{15}} \quad (13)$$

$$\frac{i_{L2}(s)}{d(s)} = \frac{-3.64 \times 10^9 \times s^3 - 9.787 \times 10^8 \times s^2 + 3.27 \times 10^{12} \times s - 3.694 \times 10^{16}}{s^4 + 3.228 \times 10^4 \times s^3 + 4.428 \times 10^8 \times s^2 + 1.8 \times 10^{12} \times s + 6.903 \times 10^{15}} \quad (14)$$

$\frac{\tilde{v}_o(s)}{d(s)}$ and $\frac{i_{L2}(s)}{d(s)}$ are not minimum phase transfer function, while $\frac{i_{L1}(s)}{d(s)}$ is minimum phase. Suggested control structure is shown in Fig. 5.

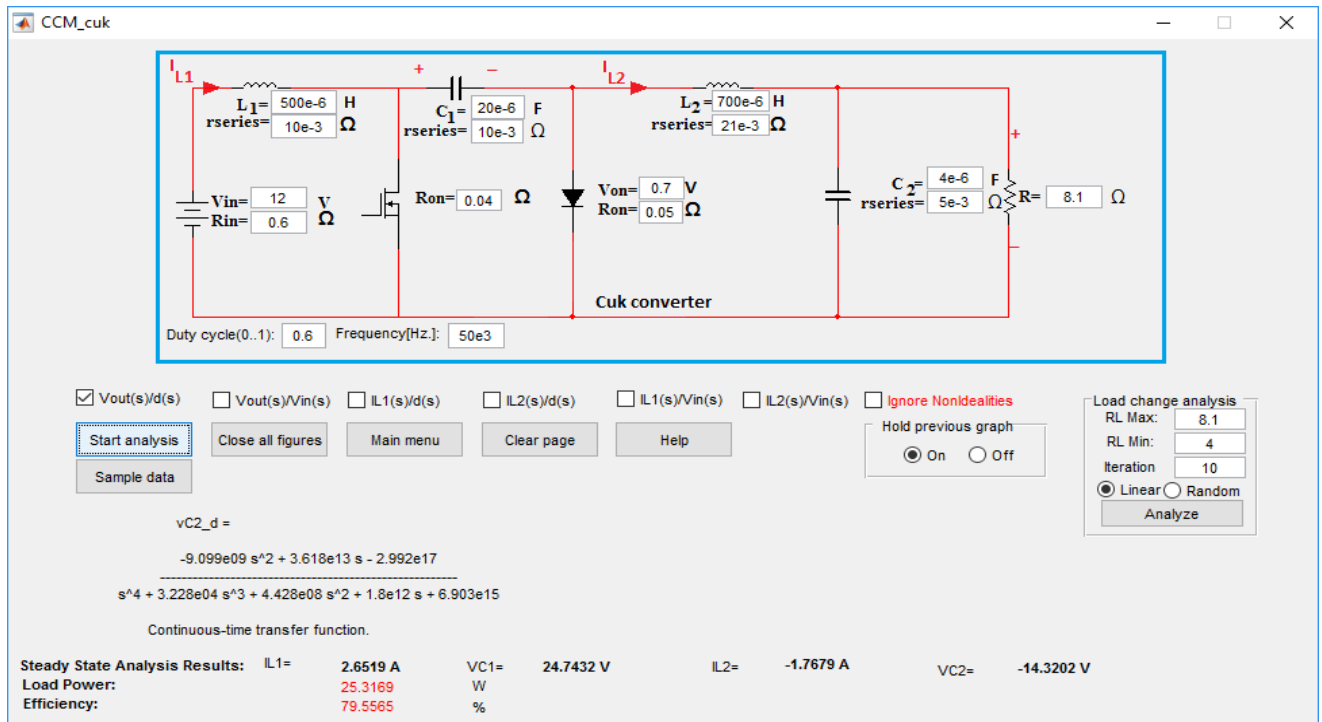


Fig. 4: Entering Cuk converter's parameters to developed software

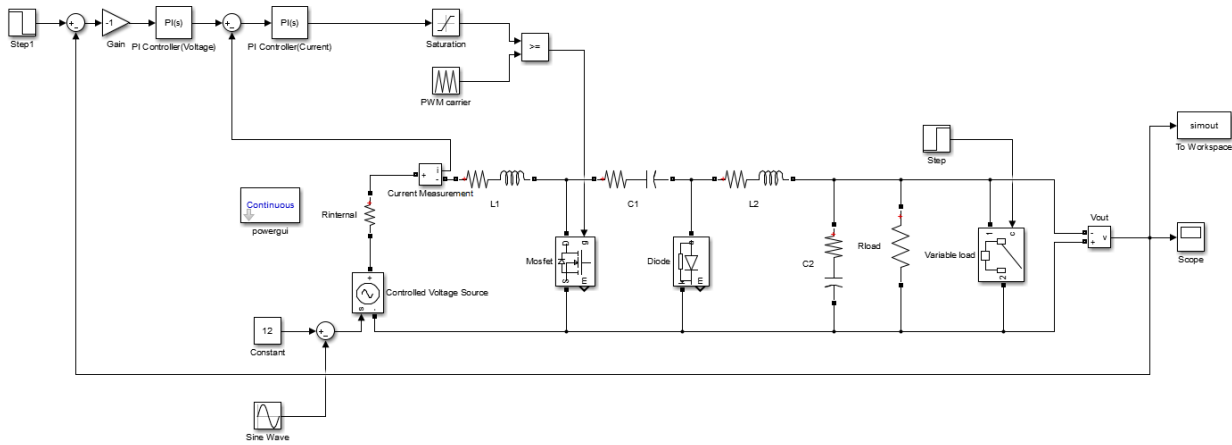


Fig. 9: Simulink diagram of Cuk converter with designed controllers

Table 1: Test scenario

Parameter's name	Time	Initial value	Final value	$\frac{Final - Initial}{Initial}$
Rload	20 ms	8.1 Ω	2.7 Ω	+67%
Vs	40 ms	12 V	6 V	+50%
Vref	80 ms	14 V	10 V	-40%

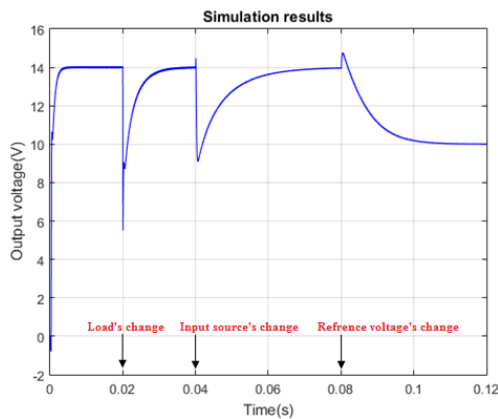


Fig. 10: Simulation results

6. Conclusion

Control theory plays an important role in power electronics. Providing a stable output voltage despite of changes in load and input voltage is not achievable without the use of control theory. Control design for converters like Cuk, SEPIC and Zeta is more challenging due to high order of converter and presence of RHP zeros. Designing a controller for Cuk converter is the aim of this paper. Converter's dynamical equations are extracted using converter dynamics toolbox for MATLAB. A controller is designed for obtained equations using MATLAB's control system toolbox. Designed controller is tested in Simulink environment. Simulation results showed the performance of designed controller.

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